The local *h*-polynomial of a triangulation of the simplex

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The local *h*-polynomial is a fundamental enumerative invariant of a triangulation Δ of a simplex, introduced by Stanley in 1992. This talk aims to review its remarkable combinatorial properties, discuss nice examples and focus on the case in which Δ is the barycentric subdivision of a triangulation of the simplex. It was shown by Stanley that the coefficients of the local *h*-polynomial of the first barycentric subdivision of the simplex of dimension *n-1* enumerate permutations of the set $\{1,2,\ldots,n\}$ without fixed points with a given number of excedances. As a special case of a more general result, we will give a combinatorial interpretation of the local *h*-polynomial of the second barycentric subdivision of a simplex of arbitrary dimension. A connection to the work of Charalambides on the enumerative combinatorics of permutations will unexpectedly appear.

References

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