Composition schemes: q-enumerations and phase transitions

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Abstract

Composition schemes are ubiquitous in combinatorics, statistical mechanics and probability theory. We give a unifying explanation to various phenomena observed in the combinatorial and statistical physics literature in the context of q-enumeration (this is a model where objects with a parameter of value k have a Gibbs measure/Boltzmann weight q^k). For structures enumerated by a composition scheme, we prove a phase transition for any parameter having such a Gibbs measure: for a critical value $q = q_c$, the limit law of the parameter is a two-parameter Mittag-Leffler distribution, while it is Gaussian in the supercritical regime ($q > q_c$), and it is a Boltzmann distribution in the subcritical regime ($0 < q < q_c$). We apply our results to fundamental statistics of lattice paths and quarter-plane walks. We also explain previously observed limit laws for pattern-restricted permutations, and a phenomenon uncovered by Krattenthaler for the wall contacts in watermelons.
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