The Expected Number of Patterns in a Random Permutation

Anant Godbole,
East Tennessee State University
godbolea@etsu.edu

Let π_n be a uniformly chosen random permutation on $[n] = \{1, 2, ..., n\}$. Using an analysis of the probability that two overlapping consecutive k-permutations are order isomorphic, the authors of Allen et al. showed that the expected number of distinct *consecutive* patterns of all lengths $k > 0$ in π_n was $\frac{n^2}{2}(1 - o(1))$, exhibiting the fact that random permutations pack consecutive patterns near-perfectly. The conjecture made in Fokuoh, that the same is true for *non-consecutive* patterns, i.e., that there are $2^n(1 - o(1))$, distinct non-consecutive patterns expected in a random permutation, is the focus of this talk. While the conjecture remains open, we make significant progress towards addressing it. Specifically, we show (using subadditivity and Fekete's lemma) that the expected number of distinct non-consecutive patterns is asymptotic to c^n for some $c \epsilon [1, 2]$, and we hope by October 2024, to reduce this gap to [1.73, 2]. Finally, the Stein-Chen method of Poisson approximation (Barbour et al.) is used to improve the main result of Allen et al. alluded to above. This is joint work with Verónica Borrás-Serrano, University of Puerto Rico at Mayagüez, Isabel Byrne, University of Delaware, Carmen Jackson, Northwestern University, Olivia LeBlanc, Colorado State University, and Nathaniel Veimau, Swarthmore College.

References

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