The distribution of the minimum observation until a stopping time, with an application to the minimal spacing in a Renewal process

Eutichia Vaggelatou[1]

[1]*Department of Mathematics, National and Kapodistrian University of Athens, Greece, evagel@math.uoa.gr*

Abstract

Let $\{X_i\}_{i\geq 1}$ be a sequence of iid random variables and $T \in \{1, 2, ...\}$ a stopping time associated with this sequence. In this paper, the distribution of the minimum observation, $\min\{X_1, X_2, ..., X_T\}$, until the stopping time T is provided by proposing a methodology based on an appropriate change of the initial probability measure of the probability space to a truncated (shifted) one on the X_i's. As an application of the aforementioned general result, the random variables $X_1, X_2, ...$ are considered to be the interarrival times (spacings) between successive appearances of events in a Renewal counting process $\{Y_t, t \geq 0\}$, while the stopping time T is set to be the number of summands until the sum of X_i's exceeds t for the first time, i.e. $T = Y_t + 1$. Under this setup, the distribution of the minimal spacing, $D_t = \min\{X_1, X_2, ..., X_{Y_t+1}\}$, that starts in the interval $[0, t]$ is investigated and a stochastic ordering relation for D_t is obtained. In addition, bounds for the tail probability of D_t are provided when the interarrival times have the IFR\DFR property. In the special case of a Poisson process, an exact formula, as well as closed form bounds and an extreme value approximation are derived for the tail probability of D_t. Furthermore, for Renewal processes with Erlang and uniformly distributed interarrival times, exact and approximation formulae for the tail probability of D_t are also proposed. Finally, numerical examples are presented to illustrate the aforementioned exact and asymptotic results and practical applications are briefly discussed.